Repetition week 10 and 11. Analysis of variance

One-way

$$Y_{ij} = \mu + \alpha_i + \varepsilon_{ij} \begin{cases} N\left(0, \sigma^2\right), \text{ and independent} \\ i = 1, 2, \ldots, \text{k}, j = 1, 2, \ldots, n_i \end{cases}$$

Hypothesis

$$H_0 = \alpha_1 = \alpha_2 = \cdots = \alpha_k = 0 \quad H_1 \text{: at least one } \alpha_i \ \neq 0.$$

Source	SS	DF	MS	F
Treatment	$SS_A = \sum_{i=1}^{k} n_i \ \bar{y}_{i.} - \bar{y}_{..} \ ^2$	k-1	$SS_A / \ k\text{-}1$	$\frac{SS_A}{k\text{-}1} \Big/ \frac{SS_E}{n\text{-}k}$
Error	$SS_E = \sum_{i=1}^{k} \sum_{j=1}^{n_i} \ y_{ij} - \bar{y}_{i.} \ ^2$	n-k	$SS_E / \ n\text{-}k$	
Total	$SS_T = \sum_{i=1}^{k} \sum_{j=1}^{n_i} \ y_{ij} \text{-} \bar{y}_{..} \ ^2$	n-1		

Randomized complete block design

$$Y_{ij} = \mu + \alpha_i + \beta_j + \varepsilon_{ij} \begin{cases} N \ \ 0, \sigma^2 \quad \text{and independent} \\ j = 1, 2, \cdots, b, \ \ i = 1, 2, \cdots, k \end{cases}$$

Hypothesis

$$H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_k = 0 \quad H_1 \text{: at least two are different from zero}$$

Sources	SS	DF	MS	F
Treatment	$SS_A = b \sum_{i=1}^{k} \ \bar{y}_{i.} - \bar{y}_{..} \ ^2$	k-1	$SS_A / \ k-1$	$\frac{SS_A}{k-1} \Big/ \frac{SS_E}{b-1 \ \ k-1}$
Block	$SS_B = k \sum_{j=1}^{b} \ \bar{y}_{.j} - \bar{y}_{..} \ ^2$	b-1	$SS_B / \ b-1$	
Error	$SS_E = \sum_{i=1}^{k} \sum_{j=1}^{b} \ y_{ij} - \bar{y}_{i.} - \bar{y}_{.j} + \bar{y}_{..} \ ^2$	(b-1)(k-1)	$SS_E / \ b-1 \ \ k-1$	
Total	$SS_T = \sum_{i=1}^{k} \sum_{j=1}^{b} \ y_{ij} - \bar{y}_{..} \ ^2$	bk-1		

Two-way analysis of variance

Model

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \ \alpha\beta \ _{ij} + \varepsilon_{ijk} \begin{cases} \quad \mathbf{N} \ \ 0,\sigma^2 \quad \text{and independent} \\ i = 1,2,\cdots,a, \ \ j = 1,2,\cdots,b, \ k = 1,2,\cdots,n \end{cases}$$

1. $H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_a = 0 \quad H_1$: at least one is different from 0.
2. $H_0 : \beta_1 = \beta_2 = \cdots = \beta_b = 0 \quad H_1$: at least one is different from 0.
3. $H_0 : \alpha\beta_{11} = \alpha\beta_{12} = \cdots = \alpha\beta_{ab} = 0 \quad H_1$: at least one is different from 0.

Sources	SS	DF	MS	F
A	$SS_A = nb\sum_{i=1}^{a} \ y_{i..} - y_{...} \ ^2$	$a-1$	$\frac{SS_A}{a-1}$	$F = \frac{MS_A}{MS_E}$
B	$SS_B = na\sum_{j=1}^{b} \ y_{.j.} - \bar{y}_{...} \ ^2$	$b-1$	$\frac{SS_B}{b-1}$	$F = \frac{MS_B}{MS_E}$
Interaction AB	$SS_{AB} = n\sum_{i=1}^{a}\sum_{j=1}^{b} \ y_{ij.} - \bar{y}_{i..} - \bar{y}_{.j.} + \bar{y}_{...} \ ^2$	$a-1 \ \ b-1$	$\frac{SS_{AB}}{a-1 \ \ b-1}$	$F = \frac{MS_{AB}}{MS_E}$
Error	$SS_E = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} \ y_{ijk} - \bar{y}_{ij.} \ ^2$	$ab \ \ n-1$	$\frac{SS_E}{ab \ \ n-1}$	
Total	$SS_T = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} \ y_{ijk} - \bar{y}_{...} \ ^2$	$abn-1$		